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Dynamic Economic Load Dispatch of Thermal Power System Using Particle Swarm Optimization Technique

By

W.M. MANSOUR* M. M. SALAMA* S. M. ABDELMAKSOU** H. A. HENRY***

Abstract:

Economic load dispatch (ELD) problem is one of the most important problems to be solved in the operation and planning of a power system. The main objective of the ELD problem is to determine the optimal schedule of output powers of all generating units so as to meet the required load demand at minimum operating cost while satisfying system equality and inequality constraints. This paper presents a new approach using Particle Swarm Optimization (PSO) for solving the ELD problem with considering the generator constraints, ramp rate limits and transmission line losses. The proposed approach has been evaluated on 26-bus, 6-unit system. The obtained results of the proposed method are compared with those obtained from the conventional lambda iteration method. The results show that the proposed approach is feasible and efficient.

Keywords:

Economic load dispatch, particle swarm optimization and ramp rate limits

* Professor of Electrical Power System, Faculty of Engineering Shoubra
** Assistant Professor of Electrical Engineering, Faculty of Engineering Shoubra
*** Demonstrator of Electrical Engineering, Faculty of Engineering Shoubra

1. Introduction:

With the development of modern power systems, economic load dispatch (ELD) problem has received an increasing attention. The primary objective of ELD problem is to minimize the total generation cost of units while satisfying all units and system equality and inequality constraints [1]. In this problem, the generation costs are represented as curves and the overall calculation minimizes the operating cost by finding the point where the total output power of the generators equals the total power that must be delivered. In the traditional ELD problem, the cost function for each generator has been represented approximately by a single quadratic function and is solved using mathematical programming based optimization techniques such as lambda iteration method, gradient method, Newton method, linear and dynamic programming methods [2,3]. All these methods assume that the cost curve is continuous and monotonically increasing. In these methods, computational time increases with the increase of the dimensionality of the ELD problem. The most common optimization techniques based upon artificial intelligence concepts such as evolutionary programming [4], simulated annealing, artificial neural networks [5], genetic algorithm [6,7], tabu search [8] and particle swarm optimization (PSO) [9-12] have been given attention by many researchers due to their ability to find an almost global optimal solution for ELD problems with operating constraints. Major problem associated with these techniques is that appropriate control parameters are required. Some times these techniques take large computational time due to improper selection of the control parameters. The PSO is a population based optimization technique first proposed by Kennedy and Eberhart in 1995. In PSO, each particle is a candidate solution to the problem. Each particle in PSO makes its decision based on its own experience together with other particles experiences. Particles approach to the optimum solution through its present velocity, previous experience and the best experience of its neighbors [13]. Compared to other evolutionary computation techniques, PSO can solve the problems quickly with high quality solution and stable convergence characteristic, whereas it is easily implemented.

2. Formulation of an ELD Problem with Generator Constraints:

The primary objective of the ELD problem is to minimize the total fuel cost of thermal power plants subjected to the operating constraints of a power system. In general, the ELD problem can be formulated mathematically as a constrained optimization problem with an objective function of the form:

$$F_T = \sum_{i=1}^n F_i(P_i) \quad (1)$$

Generally, the fuel cost function of the generating unit is expressed as a quadratic function as given in (2)

$$F_i = a_i P_i^2 + b_i P_i + c_i \quad (2)$$

The minimization of the ELD problem is subjected to the following constraints:

I) Real Power Balance Constraint:

For power balance, an equality constraint should be satisfied. The total generated power should be equal to the total load demand plus the total losses. The active power balance is given by:

$$\sum_{i=1}^n P_i = P_D + P_L \quad (3)$$

The total transmission line loss is assumed as a quadratic function of output powers of the generator units [14] that can be approximated in the form:

$$P_L = \sum_{i=1}^n \sum_{j=1}^n P_i B_{ij} P_j \quad (4)$$

II) Generator Power Limit Constraint:

The generation output power of each unit should lie between minimum and maximum limits. The inequality constraint for each generator can be expressed as:

$$P_{i, \min} \leq P_i \leq P_{i, \max} \quad (5)$$

Where $P_{i, \min}$ and $P_{i, \max}$ are the minimum and maximum power outputs of generator i (MW), respectively. The maximum output power of generator is limited by thermal consideration and minimum power generation is limited by the flame instability of a boiler.

III) Ramp Rate Limit Constraint:

The generator constraints due to ramp rate limits of generating units are given as:

a) as generation increases:

$$P_{i(t)} - P_{i(t-1)} \leq UR_i \quad (6)$$

b) as generation decreases:

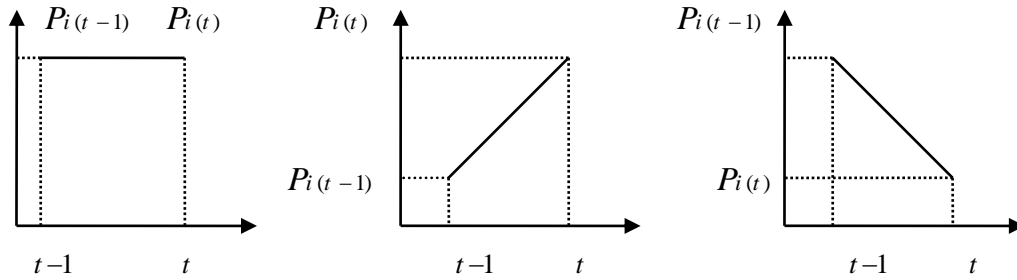
$$P_{i(t-1)} - P_{i(t)} \leq DR_i \quad (7)$$

Therefore the generator power limit constraints can be modified as:

$$\max(P_{i, \min}, P_{i(t-1)} - DR_i) \leq P_{i(t)} \leq \min(P_{i, \max}, P_{i(t-1)} + UR_i) \quad (8)$$

where $P_{i(t)}$ is the output power of generating unit i (MW) in the time interval (t) , $P_{i(t-1)}$ is the output power of generating unit i (MW) in the previous time interval $(t-1)$, UR_i is the up ramp limit of generating unit i (MW/time-period) and DR_i is the down ramp limit of generating unit i (MW/time-period).

The ramp rate limits of the generating units with all possible cases are shown in Fig.1.



Figure(1): Ramp rate limits of the generating units

3. Overview of the Lambda Iteration Method:

The formulation of Lagrange function for the ELD problem is given by:

$$F = F_T + \lambda \left(P_D + P_L - \sum_{i=1}^n P_i \right) \tag{9}$$

The condition for optimal operation can be obtained by differentiating F with respect to P_i as follows:

$$\frac{dF_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \tag{10}$$

The coordination equation can be given as:

$$f_{ii}P_i + f_i + \lambda \sum_i 2B_{ij}P_i = \lambda \tag{11}$$

The expression for output power is:

$$P_i = \frac{1 - \frac{f_i}{\lambda} - \sum_{i \neq j} 2B_{ij}P_i}{\frac{f_{ii}}{\lambda} + 2B_{ii}} \tag{12}$$

The step by step algorithm for the Lambda Iteration method is explained as follow:

Step 1: Assume a suitable value of $\lambda^{(0)}$ which has more than the largest value of

f_i (intercept of the incremental cost of various generators)

Step 2: Calculate the generations based on equal incremental production cost.

Step 3: Calculate generations at all buses ($P_1, P_2, P_3, \dots, P_i$) from equation (12).

Step 4: Check the equality and inequality constraints.

Step 5: Check if the difference in power at all generator buses between two successive iterations is less than a pre specified value (ε), if not go back to step 3.

Step 6: Calculate transmission line losses from equation (4).

Step 7: Calculate $|\Delta P| = \left| \sum_{i=1}^n P_i - P_L - P_D \right|$

If $|\Delta P| \leq \varepsilon$ (ε is the tolerance), calculate the cost of generation and the values of power for all units and then go to step 8. If $|\Delta P| > \varepsilon$, update the value of λ and go back to step 3.

Step 8: Stop

4. Particle Swarm Optimization (PSO):

Particle swarm optimization (PSO) is a population based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling. It is one of the most modern heuristic algorithms, which can be used to solve non linear and non continuous optimization problems. PSO shares many similarities with evolutionary computation techniques such as genetic algorithm (GA). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as mutation and crossover. The PSO algorithm searches in parallel using a group of random particles. Each particle in a swarm corresponds to a candidate solution to the problem. Particles in a swarm approach to the optimum solution through its present velocity, its previous experience and the experience of its neighbors. In every generation, each particle in a swarm is updated by two best values. The first one is the best solution (best fitness) it has achieved so far. This value is called P_{best} . Another best value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called g_{best} . Each particle moves its position in the search space and updates its velocity according to its own flying experience and neighbor's flying experience. After finding the two best values, the particle update its velocity according to equation (13).

$$V_i^{k+1} = \omega \times V_i^k + C_1 \times R_1 \times (P_{best}^k - P_i^k) + C_2 \times R_2 \times (g_{best}^k - P_i^k) \quad (13)$$

Where V_i^k is the velocity of particle i at iteration k , P_i^k is the position of particle i at iteration k , ω is the inertia weight factor, C_1 and C_2 are the acceleration coefficients, R_1 and R_2 are positive random numbers between 0 and 1, $Pbest_i^k$ is the best position of particle i at iteration k and $gbest^k$ is the best position of the group at iteration k .

In the velocity updating process, the acceleration constants C_1, C_2 and the inertia weight factor are predefined and the random numbers R_1 and R_2 are uniformly distributed in the range of $[0,1]$. Suitable selection of inertia weight in equation (13) provides a balance between local and global searches. Thus requiring less iteration on average to find a sufficiently optimal solution. A low value of inertia weight implies a local search, while a high value leads to global search. As originally developed, the inertia weight factor often is decreased linearly from about 0.9 to 0.4 during a run. It was proposed in [15]. In general, the inertia weight ω is set according to equation (14)

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{Iter_{\max}} \times Iter \quad (14)$$

Where ω_{\min} and ω_{\max} are the minimum and maximum value of inertia weight factor, $Iter_{\max}$ corresponds to the maximum iteration number and $Iter$ is the current iteration number.

The current position (searching point in the solution space) can be modified by equation (15)

$$P_i^{k+1} = P_i^k + V_i^{k+1} \quad (15)$$

The velocity of particle i at iteration k must lie in the range:

$$V_{i \min} \leq V_i^k \leq V_{i \max} \quad (16)$$

The parameter V_{\max} determines the resolution or fitness, with which regions are to be searched between the present position and the target position. If V_{\max} is too high, the PSO facilitates a global search and particles may fly past good solutions. Conversely, if V_{\max} is too small, the PSO facilitates a local search and particles may not explore sufficiently beyond locally good solutions. In many experiences with PSO, V_{\max} was often set at 10-20% of the dynamic range on each dimension.

The constants C_1 and C_2 represents the weighting of the stochastic acceleration terms that pull each particle toward $Pbest$ and $gbest$ positions. Low values allow particles to roam far from the target regions, while high values result in abrupt movement toward, or past, target regions. Hence, the acceleration constants were often set to be 2.0 according to past experiences.

5. Implementation of PSO for Solving ELD Problem:

A step by step procedure of the proposed PSO method for solving ELD problem is as follows:

Step 1: Select the parameters of PSO such as population size (N), acceleration constants (C_1 and C_2), minimum and maximum value of inertia weight factor (ω_{\min} and ω_{\max}).

Step 2: Initialize a population of particles with random positions and velocities. These initial particles must be feasible candidate solutions that satisfy the practical operation constraints.

Step 3: Evaluate the fitness value of each particle in the population using the objective function given in equation (2).

Step 4: Compare each particle's fitness with the particles P_{best} . If the current value is better than P_{best} , then set P_{best} equal to the current value.

Step 5: Compare the fitness with the population overall previous best. If the current value is better than g_{best} , then set g_{best} equal to the current value.

Step 6: Update the velocity of each particle according to equation (13).

Step 7: The position of each particle is modified using equation (15).

Step 8: Go to step 9 if the stopping criteria is satisfied, usually a sufficiently good fitness or a maximum number of iterations. Otherwise go to step 3.

Step 9: The particle that generate the latest g_{best} is the optimal generation power of each unit with the minimum total cost of generation.

6. Case Study and Simulation Results:

To verify the effectiveness of the proposed particle swarm optimization (PSO) algorithm, a six unit thermal power generating plant was tested. The proposed algorithm has been implemented in MATLAB software. The proposed algorithm is applied to 26 buses, 6 generating units with generator constraints, ramp rate limits and transmission losses [16]. The results obtained from the proposed PSO method will be compared with the outcomes obtained from the conventional lambda iteration method in terms of the solution quality and computation efficiency. The fuel cost data and ramp rate limits of the six thermal generating units were given in Table 1. The load demand for 24 hours is given in Table 2. B-loss coefficients of six units system is given in Equation (17). Output powers, power loss and total fuel cost obtained by the lambda iteration method for 24 hours are given in Table 3. Output powers, power loss and total fuel cost obtained

by the proposed PSO method for all power demands are given in Table 4. Figure (2) to figure (7) show the relation between fuel cost of each unit and 24 hours by the two used methods. When the Lambda iteration method is used to solve this system, it has been observed that the minimum cost curve converges within the range of 1500 - 2000 iterations while in PSO technique the cost curve converge within the range of 20-40 iterations. So the computational time of the proposed PSO method is much less than the Lambda iteration method.

Some parameters must be assigned for the use of PSO to solve ELD problems as follows:

- Population size = 20
- Maximum number of iterations = 120
- Acceleration constants $C_1 = 2.0$ and $C_2 = 2.0$
- Inertia weight parameters $\omega_{max} = 0.9$ and $\omega_{min} = 0.4$

$$B_{ij} = 10^{-3} \begin{bmatrix} 1.7 & 1.2 & 0.7 & -0.1 & -0.5 & -0.2 \\ 1.2 & 1.4 & 0.9 & 0.1 & -0.6 & -0.1 \\ 0.7 & 0.9 & 3.1 & 0.0 & -1.0 & -0.6 \\ -0.1 & 0.1 & 0.0 & 0.24 & -0.6 & -0.8 \\ -0.5 & -0.6 & -0.1 & -0.6 & 12.9 & -0.2 \\ -0.2 & -0.1 & -0.6 & -0.8 & -0.2 & 15.0 \end{bmatrix} \quad (17)$$

Table (1): Fuel cost coefficients and ramp rate limits of six-units thermal power system

Unit	a_i (\$/MW ²)	b_i (\$/MW)	c_i (\$)	$P_{i, min}$ (MW)	$P_{i, max}$ (MW)	UR_i (MW/H)	DR_i (MW/H)
1	0.0070	7.0	240	100	500	80	120
2	0.0095	10.0	200	50	200	50	90
3	0.0090	8.5	220	80	300	65	100
4	0.0090	11.0	200	50	150	50	90
5	0.0080	10.5	220	50	200	50	90
6	0.0075	12	190	50	120	50	90

Table (2): Load demand for 24 hours of six-units thermal power system

Time (H)	Load Demand (MW)	Time (H)	Load Demand (MW)	Time (H)	Load Demand (MW)	Time (H)	Load Demand (MW)
1	955	7	989	13	1190	19	1159
2	942	8	1023	14	1251	20	1092
3	935	9	1126	15	1263	21	1023
4	930	10	1150	16	1250	22	984
5	935	11	1201	17	1221	23	975
6	963	12	1235	18	1202	24	960

Table (3): Output powers, power losses and total fuel cost for 24 hours by lambda iteration method of 6-units power system

Time (H)	P ₁ (MW)	P ₂ (MW)	P ₃ (MW)	P ₄ (MW)	P ₅ (MW)	P ₆ (MW)	Loss (MW)	Total Fuel Cost (\$)
1	380.4615	123.3371	209.6713	86.0619	112.0658	50.0000	6.5454	11411.49
2	377.2356	120.9510	207.2177	83.3767	109.5253	50.0000	6.3770	11247.53
3	375.5586	119.7120	205.9428	81.9769	108.1951	50.0000	6.2903	11162.39
4	374.2797	118.7655	204.9696	80.9146	107.1906	50.0000	6.2249	11097.72
5	375.5586	119.7120	205.9428	81.9769	108.1951	50.0000	6.2903	11162.39
6	382.4757	124.8263	211.2028	87.7407	113.6547	50.0000	6.6520	11514.21
7	388.7815	129.4879	215.9959	93.0021	118.6308	50.0000	6.9922	11837.37
8	396.2380	134.9471	221.5851	99.1484	124.3931	54.1809	7.3811	12271.75
9	417.6234	150.5209	237.4881	116.7043	140.7398	71.6249	8.5625	13601.46
10	422.5871	154.1345	241.1754	120.7834	144.5216	75.6584	8.8582	13914.01
11	433.2687	161.9092	249.1052	129.5670	152.6439	84.3188	9.5220	14591.57
12	440.2961	167.0206	254.3159	135.3508	157.9794	90.0073	9.9789	15041.12
13	430.9261	160.2043	247.3667	127.6400	150.8645	82.4217	9.3732	14442.39
14	443.6691	169.4772	256.8197	138.1265	160.5320	92.7258	10.2039	15257.84
15	446.1478	171.2806	258.6574	140.1676	162.4089	94.7258	10.3716	15417.58
16	443.4625	169.3269	256.6666	137.9565	160.3755	92.5590	10.1900	15244.54
17	437.4019	164.9171	252.1718	132.9678	155.7812	87.6628	9.7888	14855.58
18	433.4079	162.0080	249.2058	129.6825	152.7530	84.4367	9.5309	14600.51
19	424.4496	155.4879	242.5559	122.3154	145.9427	77.1755	8.9713	14031.72
20	410.5219	145.3478	232.2076	110.8721	135.3244	65.8493	8.1534	13156.92
21	396.2380	134.9471	221.5851	99.1484	124.3931	54.1809	7.3811	12271.75
22	387.5739	128.5950	215.0780	91.9947	117.6794	50.0000	6.9263	11775.33
23	385.4290	127.0100	213.4481	90.2030	115.9833	50.0000	6.8100	11665.24
24	381.6678	124.2280	210.5879	87.0701	113.0232	50.0000	6.6092	11473.07
Total Generation Cost (\$)								313045.50

Table (4): Output powers, power losses and total fuel cost for 24 hours by particle swarm optimization (PSO) of 6-units power system

Time (H)	P₁ (MW)	P₂ (MW)	P₃ (MW)	P₄ (MW)	P₅ (MW)	P₆ (MW)	Loss (MW)	Total Fuel Cost (\$)
1	381.5714	120.8702	210.4459	86.5109	112.1413	50.0000	6.5396	11410.86
2	375.6001	118.3435	208.2801	84.9338	111.2008	50.0000	6.3583	11248.50
3	372.1313	116.8757	207.0221	84.5696	110.6544	50.0000	6.2531	11161.44
4	369.6539	115.8275	206.1235	84.3094	110.2642	50.0000	6.1785	11099.41
5	372.1313	116.8757	207.0221	84.5696	110.6544	50.0000	6.2531	11161.44
6	384.9939	122.2077	211.6352	87.7447	113.0670	50.0000	6.6485	11511.17
7	394.9573	126.2967	215.8115	92.0565	116.8847	50.0000	7.0069	11838.94
8	399.0251	133.7940	222.1091	96.2167	122.7501	56.4862	7.3813	12270.52
9	420.7399	145.6516	239.2728	114.8466	140.7699	73.2912	8.5720	13599.88
10	427.7292	148.1256	243.1505	118.8350	143.3025	77.7136	8.8569	13914.45
11	443.1065	154.9287	247.8817	127.5496	151.0744	85.9687	9.5097	14588.85
12	452.3793	160.5090	251.5363	133.1629	155.4444	91.9242	9.9565	15042.84
13	439.1911	153.2747	246.5499	125.5640	150.2959	84.4946	9.3702	14442.65
14	456.1396	162.7014	254.3155	136.3330	157.9999	93.6931	10.1835	15257.49
15	458.8923	164.5063	255.7054	138.9330	159.2996	95.9928	10.3293	15419.10
16	455.6563	162.5096	254.1511	136.2857	157.9286	93.6342	10.1662	15244.01
17	447.6668	158.8386	250.4331	129.9988	153.7482	90.0963	9.7821	14855.29
18	443.5074	155.0878	248.0181	127.6891	151.1336	86.0866	9.5234	14602.16
19	430.7223	149.6136	244.1688	120.3298	144.2447	78.8929	8.9723	14032.85
20	414.3692	141.9872	233.4176	109.2408	133.8613	67.2768	8.1531	13157.51
21	399.0251	133.7940	222.1091	96.2167	122.7501	56.4862	7.3813	12270.52
22	393.7541	125.3609	214.9522	90.9498	115.9247	50.0000	6.9416	11775.78
23	390.2814	124.3894	213.6910	89.0846	114.3769	50.0000	6.8235	11662.16
24	383.5108	121.5776	211.0951	87.4883	112.9324	50.0000	6.6042	11473.52
Total Generation Cost (\$)								313041.40

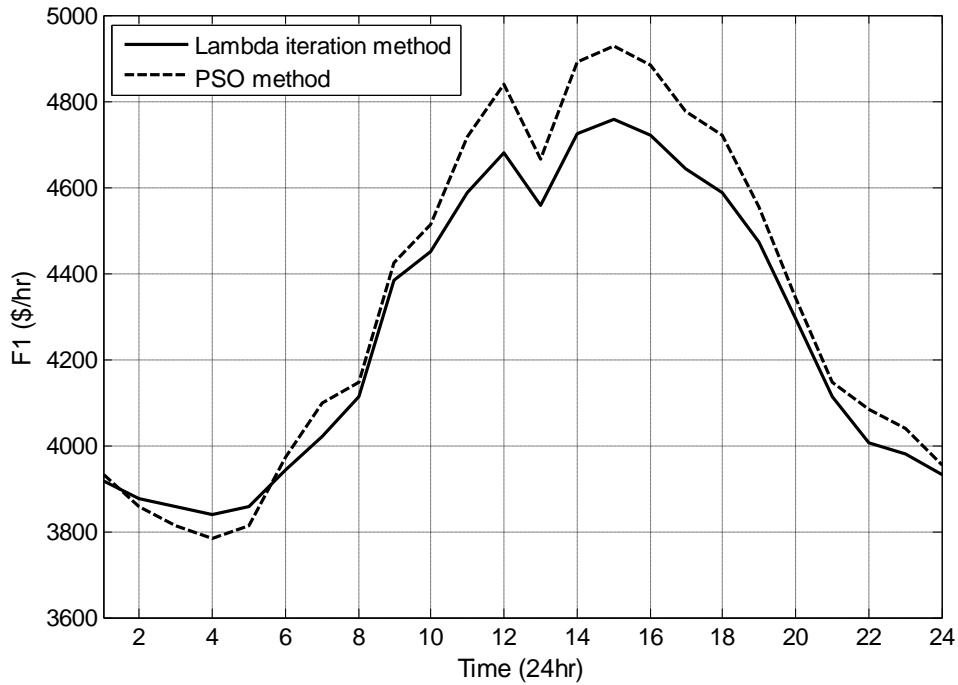


Figure (2): Fuel cost of unit 1 versus 24 hour by using two methods

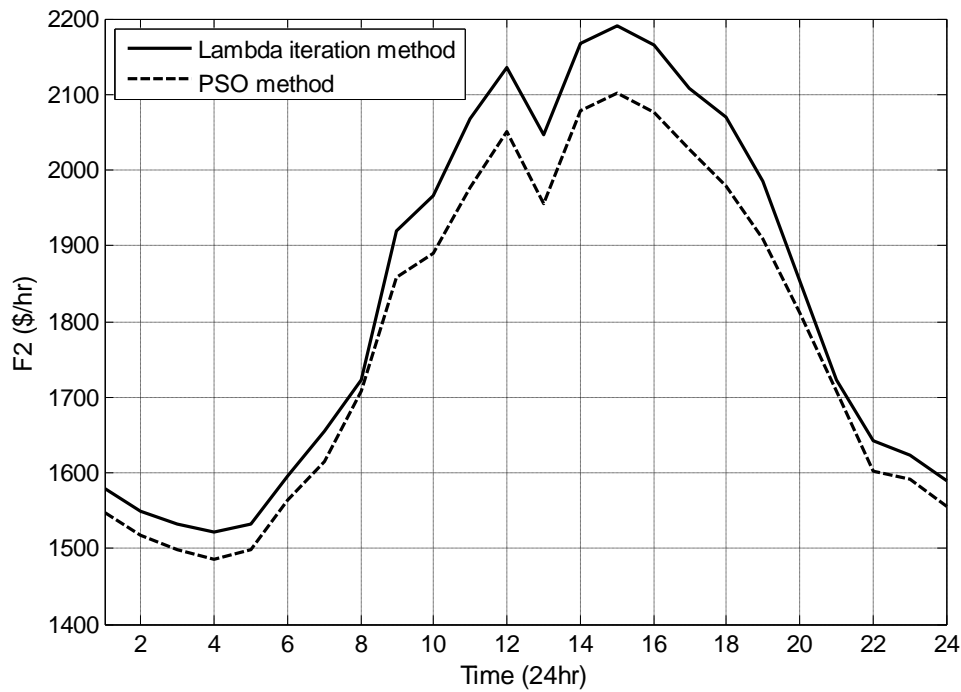


Figure (3): Fuel cost of unit 2 versus 24 hour by using two methods

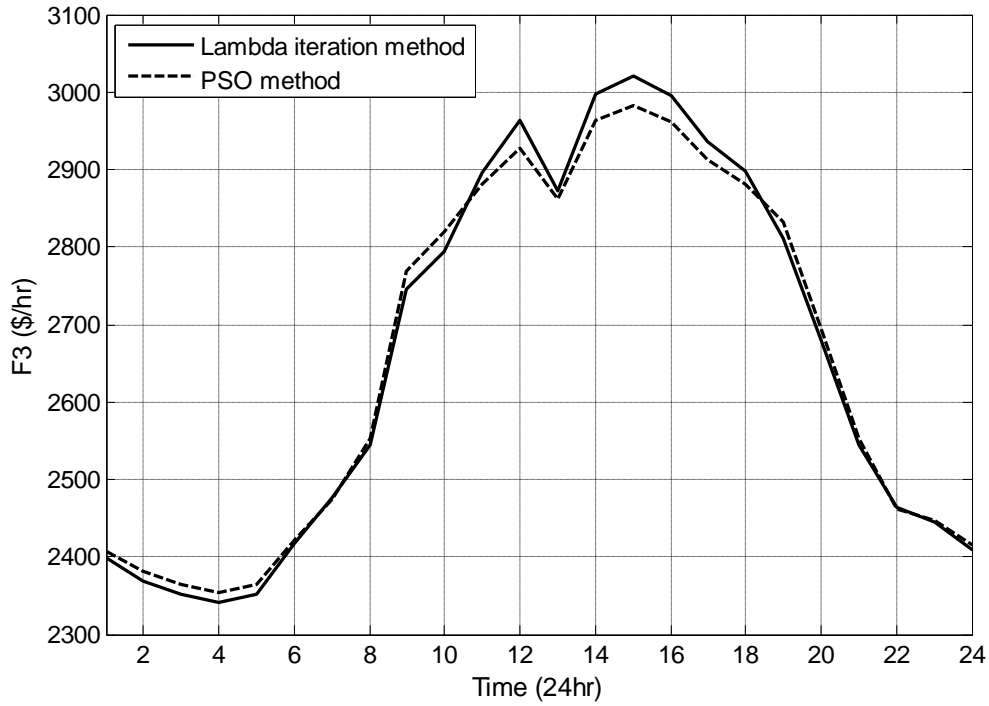


Figure (4): Fuel cost of unit 3 versus 24 hour by using two methods

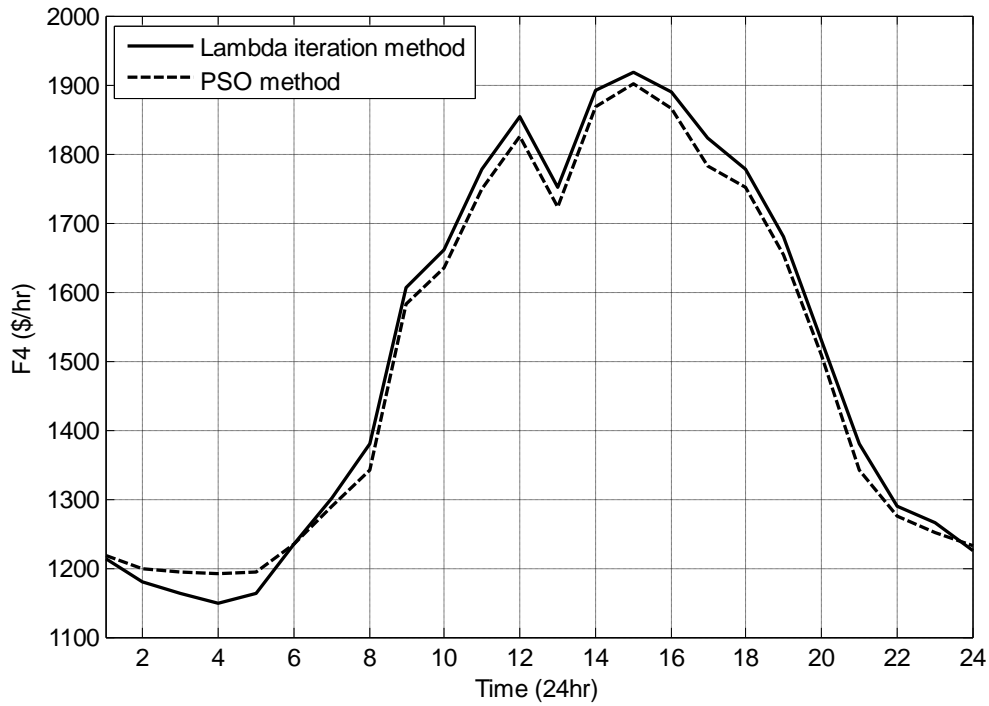


Figure (5): Fuel cost of unit 4 versus 24 hour by using two methods

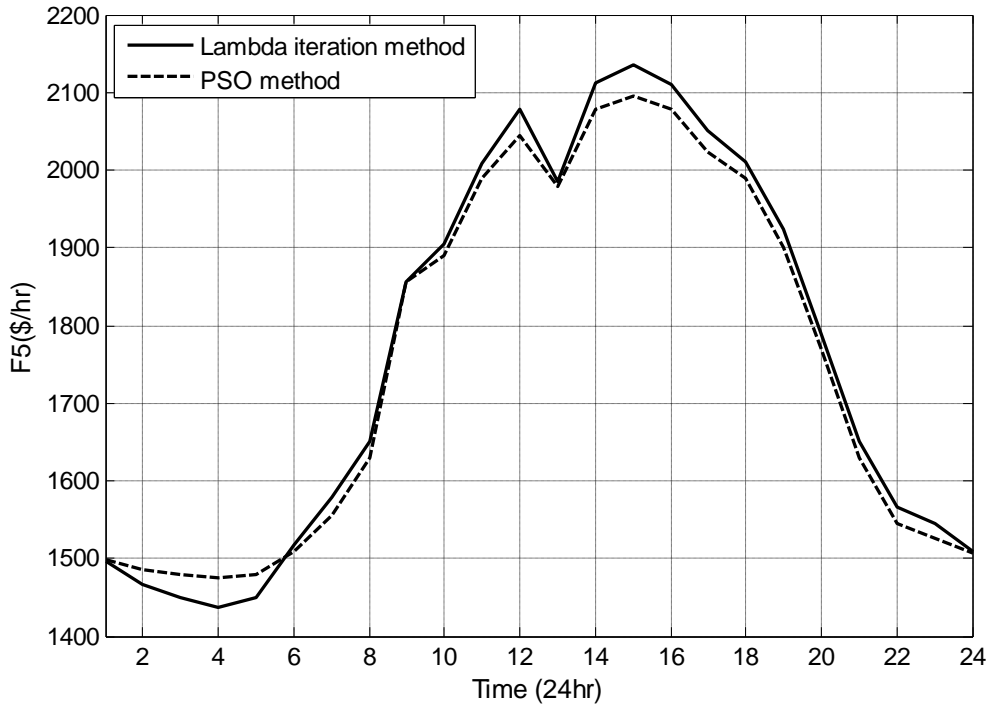


Figure (6): Fuel cost of unit 5 versus 24 hour by using two methods

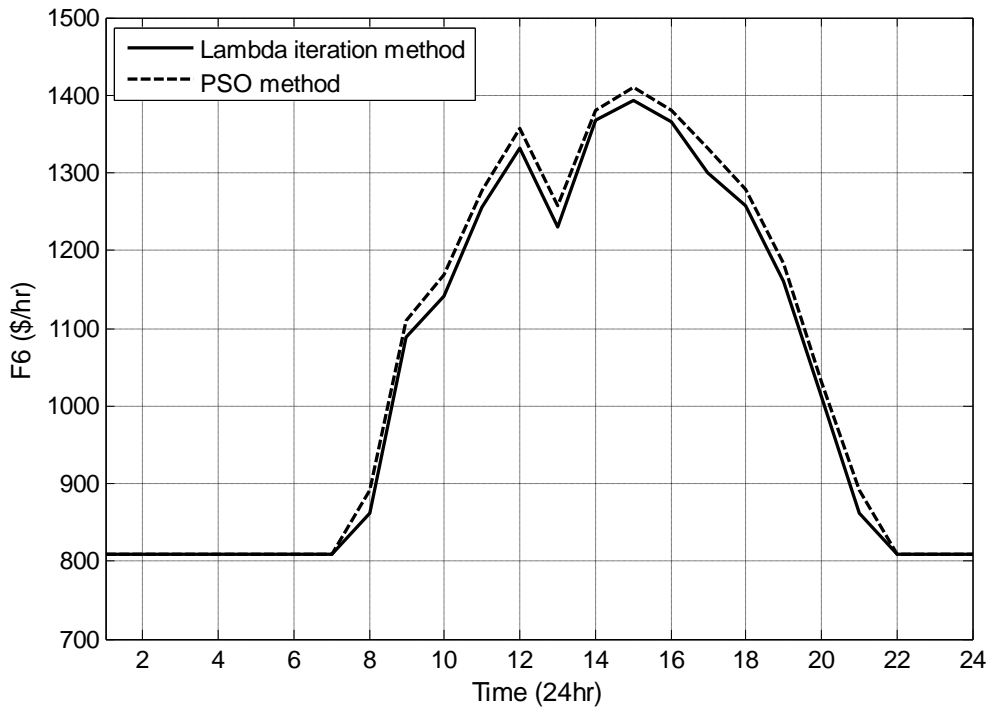


Figure (7): Fuel cost of unit 6 versus 24 hour by using two methods

7. Conclusions:

In this paper, particle swarm optimization (PSO) algorithm is used to solve the ELD problem. The proposed PSO algorithm has been successfully implemented for solving the ELD problem of a power system consists of 6 units with different constraints such as real power balance, generator power limits and ramp rate limits. From the tabulated results, it is clear that the PSO algorithm gives high quality solutions with fast convergence characteristic compared to the lambda iteration method. The PSO algorithm performs better than lambda iteration method in terms of the power loss. The lambda iteration method is also applicable, but it can converge to the minimum generation cost after so many iterations. So, the computational time of the lambda iteration method is much greater than the proposed PSO algorithm.

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Nomenclatures:

- n ... Total number of generating units
- $F_i(P_i)$... Operating fuel cost of generating unit i in (\$/hr)
- F_T ... Total fuel cost of the system in (\$/hr)
- P_i ... Real output power of unit i in (MW)
- α_i, b_i and c_i ... Fuel cost coefficients of generating unit i
- P_D ... Total load demand in (MW)
- P_L ... Total transmission line losses in (MW)
- B_{ij} ... Transmission loss coefficients matrix